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# Total tree and merchantable stem biomass equations for Ontario hardwoods

I.S. Alemdag

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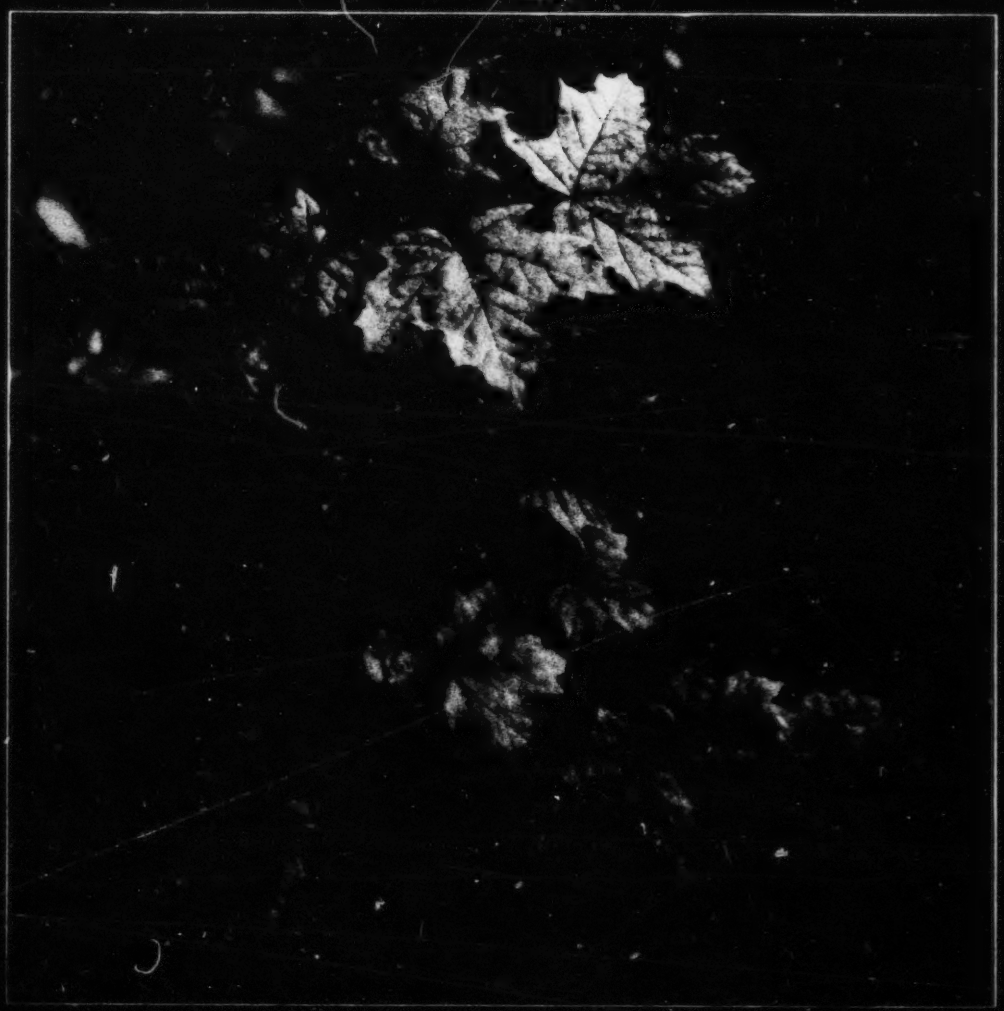
Petawawa National Forestry Institute



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Through the FORSTATS program, which involves all regional establishments of the Canadian Forestry Service, the FSSB coordinates the acquisition and publication within the CFS of national statistics on the forest of Canada.

Every five years, the FSSB publishes Canada's Forest Inventory; the official report on the location, extent, species, and condition of the forest resource. In addition, the FSSB is working closely with the provinces to expand the information available on changes to the forest from fire, harvesting, insects and disease, and from forest management activities. This information is essential to the development of sound policies for the improved management of this important and renewable natural resource.

# TOTAL TREE AND MERCHANTABLE STEM BIOMASS EQUATIONS FOR ONTARIO HARDWOODS

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I.S. Alemdag

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## Abstract

Aboveground biomass equations for single trees of 19 hardwood species of Ontario were developed. These equations are for estimating biomass, (a) in terms of ovendry mass for the main components of trees based on diameter at breast height and tree height, and (b) in terms of percent of the total stem mass for the merchantable and unmerchantable portions of the stem based on either breast height diameter and merchantable diameter, or tree height and merchantable height. In addition, several other biomass relationships were established. Computer produced tables for preparing the data for analysis are included in this report, and application of the prediction equations demonstrated.

## Résumé

Des équations ont été établies pour la biomasse de la partie épigée d'arbres individuels de 19 espèces feuillus de l'Ontario. Ces équations permettent d'estimer: (a) la biomasse anhydre des principales composantes des arbres à partir du diamètre à hauteur de poitrine et de la hauteur de l'arbre; et (b) le pourcentage de la masse des parties marchandes et non marchandes de la tige par rapport à la masse totale de la tige, soit à partir du diamètre à hauteur de poitrine et du diamètre marchand, ou de la hauteur de l'arbre et de la hauteur marchande. En outre, plusieurs autres relations pour la biomasse ont été établies. On trouvera dans le rapport des tables produites par ordinateur servant à préparer les données pour l'analyse, et une démonstration de l'application des équations.

# TOTAL TREE AND MERCHANTABLE STEM BIOMASS EQUATIONS FOR ONTARIO HARDWOODS

## INTRODUCTION

Biomass equations are required for the direct estimation of biomass values for the main components of trees as well as of the merchantable and unmerchantable portions of stems in a forest inventory, or in any operation requiring the biomass of individual trees or stands. Such equations have been developed for some Ontario hardwoods in earlier studies (Alemdag 1981, 1982; Alemdag and Horton 1981). The present study covers the remaining major hardwood species of Ontario. Aboveground biomass estimation equations of 19 hardwoods are presented, including those of previously published species for the convenience of the reader. A species list can be found in Appendix A.

## METHODS

The methods used in this study follow Alemdag (1981, 1982) and Alemdag and Horton (1971). Tree measurement and processing of wood samples for the aboveground biomass were conducted following the guidelines provided in a manual prepared for this purpose (Alemdag 1980). These field and laboratory procedures can be found in Appendix B.

### Data

In addition to data on the four tree species from earlier studies, 1061 living sample trees were collected from 15 tree species from various localities in Ontario[1] and all data merged. Of these trees, 993 were from tree sizes of 5.1 cm and larger in outside-bark diameter at breast height of 1.30 m (d), and 68 from the smaller sizes. These trees were so distributed as to cover the existing tree height (h) classes of each species. Summaries of these data can be found in Table 1 (Appendix D) and in Table 6 (Appendix E). Figure 1 illustrates the various tree components for which data were collected.

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[1]This work was performed in 1982 and 1983 by the Dendron Resource Surveys Limited, 880 Lady Ellen Place, Ottawa, Ontario, K1Z 5L9 under a Canadian Forestry Service ENFOR contract.

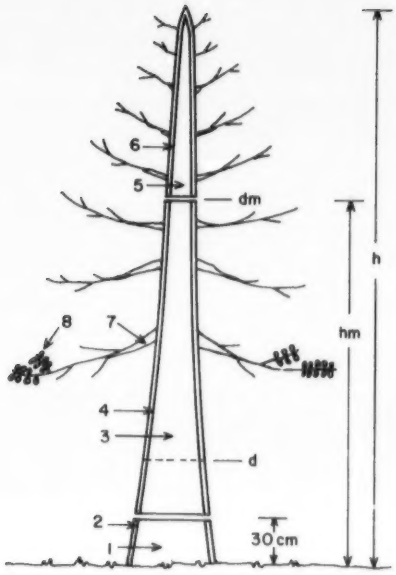


Figure 1. Diagramatic presentation of various components of trees (1) stump wood, (2) stump bark, (3) net merchantable stem wood, (4) net merchantable stem bark, (5) top wood, (6) top bark, (7) live branches, (8) twigs plus leaves.

**Compilation**

Following the preparation of data, basic compilations were done with regard to mass, wood density, and volume calculations. Details of these procedures can be found in Appendix B. Some computer processed tables related to these compilations are provided in Appendix C.

**Analysis**

After the living-tree data were assembled, analyses were conducted towards developing ovendry mass (OM) estimation equations for the main tree components (stem wood, stem bark, live branches, twigs plus leaves), and for the merchantable and unmerchantable portions of the stem (merchantable stem wood, merchantable stem bark, top wood plus bark) with stump wood and bark included in the merchantable section. These analyses were confined only to data of the most recently collected tree species. Main-component equations of largetooth aspen were taken from Alemdag and Horton (1981), and of ironwood from Alemdag (1981). Trembling aspen and white birch, for which main tree component equations had been prepared earlier for two forest regions in Ontario (Alemdag and Horton 1981), were recalculated without regional separation. Merchantability factors for trembling aspen, largetooth aspen, and white birch were adopted from Alemdag (1982). No merchantable analysis was made for ironwood because of the unavailability of merchantable-size trees of this species.

As discussed in detail in the two earlier studies (Alemdag 1981, Alemday and Horton 1981), the following model was preferred for direct estimation of the ovendry masses of the main tree components and of the whole tree as well as of dead branches:

$$OM = b_1 \cdot d^2 h \qquad (1)$$

where OM, d, and h are as defined above. This model has been used by various researchers for estimating stem volume, and is referred to as the constant form-factor model by Spurr (1952) and by Clutter et al. (1983). In the present study the model was used for trees with d equal to and larger than 5.1 cm, for each species separately. This model satisfies the additivity of the OM of main components to equal the OM of the whole tree when the latter is estimated independently, and eliminates the risk of having negative estimates with small trees. After these through-the-origin equations were developed, an analysis was conducted with regard to the bias which may occur in their application, employing the whole tree equation of the combined species as an example.

For trees with d between 0.1 cm and 5.0 cm, to obtain ovendry mass of the whole tree of all species combined without separating the main tree components, the following model was employed:

$$OM = b_0 + b_1 \cdot d^2 h \qquad (2)$$

Ovendry mass of seedlings (woody plants under 1.30 m of height) was determined by classifying them within three height classes, and taking the averages of each for all tree species combined and for the whole tree.

Ovendry mass of merchantable and unmerchantable components of a stem were studied indirectly, that is, no direct mass-estimation equation was developed for these components. Instead, they were expressed as percent of the total stem mass of wood and bark, as factors to be used later with a known ovendry mass value of total stem. This procedure, discussed in detail in an earlier study (Alemdag 1982), provides accurate solutions to the problem. Following the same method, in the present study, the above mentioned merchantable and unmerchantable stem components were analyzed for each tree species, either for a merchantable top diameter (dm) or for a merchantable height (hm). The models used were in the following forms:

$$OM\% = b_0 + b_1 \cdot (dm/d) + b_2 \cdot (dm/d)^2 \qquad (3)$$

$$OM\% = b_0 + b_1 \cdot (hm/h) + b_2 \cdot (hm/h)^2 \qquad (4)$$

where OM% is ovendry mass percentage of wood or bark of merchantable section below a given diameter or height, or ovendry mass percentage of wood plus bark of tree top above the same given diameter or height. The other variables are as stated before. To determine the net merchantable percentage of the stem wood and stem bark by deduction, their percentages of the total stem mass were calculated for different stump heights up to 30 cm. This approach was necessary because mass of stump wood and stump bark had been included in the merchantable portion of the stem. Calculation of the percentage mass distribution within the stump considered the latter as the frustum of a neiloid (Alemdag 1982).

Equations of the main tree components and of merchantability factors were developed by regression analysis. The goodness of fit and the regression accuracy were expressed by the coefficient of determination ($r^2$) or multiple

determination ($R^2$), and by the standard error of estimate as a percent of the mean (SEE%).

In addition to these analyses, for each species, the ovendry mass/green mass (OM/GM) ratios of each tree component and of the whole tree were calculated using the sums of the observed mass values. Also, the percentage of the ovendry mass of tree components in relation to the ovendry mass of stem wood were calculated based on the above developed mass estimation equations. Furthermore, the average basic wood density was established for each tree species by taking the arithmetic average of the basic wood densities of the sample trees.

After analysing individual tree species, all the above analyses were conducted once for all the 19 hardwood species combined.

## RESULTS

The regression analysis for trees equal to or larger than 5.1 cm of d, with respect to the estimation of the ovendry mass of main tree components, whole tree, and dead branches, resulted in establishing the $b_1$ coefficients of Equation 1 for each species and the species combinations. These regression coefficients together with some statistical data are provided in Table 2 of Appendix D. As is noted, the sum of the component coefficients is equal to the coefficient of the whole tree, with dead branches excluded. The examination of bias conducted on the accuracy of these equations produced encouraging results (Appendix F). This test was performed with the same data used to derive these prediction equations, and only for the whole tree mass of combined species. It was found, in all $d^2h$ classes except one, that bias ranged from -5% to +4% with zero biases in five classes. For all classes combined the bias was only -0.36%.

For the trees with a d from 0.1 cm to 5.0 cm, for all species together and for the whole tree mass only, the coefficients of Equation 2 were established as being $b_0 = 0.600$ and $b_1 = 0.020294$. For the seedlings of all species combined, the average ovendry mass of the whole tree for the three height classes were found to be as follows:

Stems from 0.01 m to 0.30 m of height = 0.009 kg

Stems from 0.31 m to 0.80 m of height = 0.060 kg

Stems from 0.81 m to 1.30 m of height = 0.298 kg

Percentage values of the component ovendry masses in the stem wood ovendry mass are presented in Table 3 for each species. Ovendry mass/green mass ratios, which were established under weather conditions at the time of data collection are given in Table 4. Average basic wood densities of each species are provided in Table 5.

For the merchantable size trees, analyses resulted in establishing the regression coefficients of Equations 3 and 4 for the estimation of percents of the merchantable and unmerchantable portions of the total stem. These coefficients, together with the necessary statistical information, are

presented in Tables 7 and 8 (Appendix E). Table 7 has to be used with Equation 3 when merchantability is defined by the merchantable top diameter and Table 8 with Equation 4 when merchantability is defined by merchantable height. It should be noted that the sum of these percentages of merchantable stem wood, merchantable stem bark and top wood plus bark is 100%, the total stem wood plus bark. This procedure will yield gross merchantable stem wood and gross merchantable stem bark masses. For the net merchantable values above 30 cm stump height, the average stump percentage deductions are given in Table 9 for each species. Mass distribution within the stump, as percent of the total stump mass at 30 cm is provided in Table 10.

In developing the merchantable factors, because of the limited field data collected at the very top of the stems and because of the nature of the second degree polynomials, there is an unavoidable situation that the user of these factors should be aware of. This is that, as explained in an earlier study (Alemdag 1982), below a restricted dm/d ratio (Equation 3) or above a restricted hm/h ratio (Equation 4), calculations result in unacceptable percentage values. Therefore, it is not permissible to use these equations for the out-of-confined ratios, and for this reason, the permitted ratios of dm/d and hm/h were calculated for each species and are given in Table 12.

## APPLICATION

A biomass inventory or a calculation of the ovendry mass of individual living trees or stands will involve the above estimation equations applied in the following manner:

1. Calculation of main tree components:

(a) When calculating the ovendry masses of individual trees it is necessary to have d and h of these trees, either directly measured or estimated, to be employed with Equation 1 (Table 2). If calculations are to be made on an area basis, either the sum of individual trees can be taken or the mean stand d and mean stand h can be used with this equation. The result has to be multiplied by the number of trees per area--this could be done for diameter classes as well. For example, a white birch tree with a d = 24.0 cm and h = 20.00 m contains 186.8 kg of stem wood, 33.1 kg of stem bark, 40.6 kg of live branches, and 9.9 kg of twigs plus leaves; a total of 270.4 kg for whole tree ovendry mass. In the same manner, Equation 2 has to be used for the trees under 5.1 cm of d. If required, seedlings can be calculated by the three height classes.

(b) If d and h values of the individual trees or mean values of d and h of a stand are not available but the total stem volume inside bark is known, first, the ovendry mass of stem wood can be calculated using this volume and the species' basic wood density (Table 5). Then, the ovendry masses of the other tree components have to be calculated using component percentages provided in Table 3. For example, a pure white birch stand with an inside-bark volume of 200.0 $m^3$/ha will have 107 200 kg/ha of stem wood, 18 974 kg/ha of stem bark, 23 262 kg/ha of live branches, 5 682 kg/ha of twigs plus leaves, and 155 118 kg/ha of whole tree ovendry mass.

(c) When an inventory is conducted using the point sampling method, the ovendry mass of tree components on a sample point will be determined by employing a modified form of Equation 1. This form is $OM/ha = (40\ 000 \cdot b_1/\pi) \cdot G \cdot h$, where G is the per hectare value of basal area in terms of $m^2/ha$. For instance, in a pure white birch stand of G = 25.0 $m^2/ha$ and mean stand h = 16.00 m, the stand will contain 82 562 kg/ha of stem wood, 14 632 kg/ha of stem bark, 17 953 kg/ha of live branches, 4 375 kg/ha of twigs plus leaves, and 119 522 kg/ha of the whole tree ovendry mass.

2. Calculation of merchantable and unmerchantable components of stem:

When tree dimensions of d and h of individual trees or the mean values of these variables for stands are available, either by direct measurements or by estimations, the requirement for proceeding with the calculations is to know either the allowable merchantable top diameter or merchantable height. Thus:

(a) If merchantable top diameter is specified, first, Equation 3 (Table 7) will be used in order to calculate the percentage values of gross merchantable sections and of unmerchantable section. Secondly, these percentages will be applied to the ovendry mass of stem wood plus bark to arrive at the actual masses. For example, the same white birch tree of 1(a), with dm = 10.2 cm, will have 82.29% merchantable stem wood, 14.21% merchantable stem bark, and 3.50% top wood plus bark. In terms of ovendry mass these are 181.0 kg, 31.2 kg, and 7.7 kg, respectively.

(b) If merchantable height is given as a specification of merchantability, the same procedure as above (a) will be followed, but this time using Equation 4 (Table 8). For instance, the same white birch tree used in 1(a), when defined by its hm being equal to 12.50 m (five 2.50 m logs), will contain 77.97% or 171.5 kg merchantable stem wood, 13.02% or 28.6 kg merchantable stem bark, and 9.01% or 19.8 kg top wood plus bark.

(c) Since these merchantable stem wood and bark values contain stump wood and bark, a deduction procedure should apply in order to arrive at the net merchantable figures. Let us assume that the above given tree was cut at 20 cm stump height. Employing average stump percentages of white birch (Table 9) and ovendry mass percentage distribution of various stump heights (Table 10) it will be found that, at 20 cm stump height, stump wood is 3.53% (= 5.16% x 68.36%) and stump bark is 0.60% (= 0.88% x 68.36%) of the total stem wood plus bark (Table 11). Thus, in the example of 2(a), net merchantable stem wood is 78.76% or 173.2 kg and net merchantable stem bark is 13.61% or 29.9 kg; and in the example of 2(b) these are 74.44% or 163.7 kg and 12.42% or 27.3 kg, respectively. Another example is given in Table 13.

Table 11, which shows for white birch, the deduction percentages of stump wood and of stump bark at different stump heights up to 30 cm, can easily be prepared for the other tree species in the same way, using figures given in Tables 9 and 10.

When using Equations 3 and 4, the limitations specified in Table 12 should be given serious consideration: ratios respectively below and above these permitted values should not be employed with these equations.

3. Calculation of logging residues:

As an example, let us take the shortwood harvesting system. In a logging operation of this type it is assumed that live branches, twigs plus leaves, top wood plus bark, and stump wood plus bark are left in the forest, and wood and bark of the net merchantable portion of the stem is taken to the mill. If we assume that the ovendry mass of tree top above a given merchantable diameter, and of the stump, are calculated in connection with the total stem mass, then the formula to be used for the ovendry mass of the logging residues will be in the form of

$$OM = d^2h \cdot (a_1 + a_2 \cdot (a_3 + a_4 \cdot (dm/d) + a_5 \cdot (dm/d)^2 + k \cdot q))$$

where the new coefficients are as follows:

$a_1$ = $b_1$ of live branches plus $b_1$ of twigs plus leaves of Equation 1 (Table 2),

$a_2$ = $b_1$ of stem wood plus $b_1$ of stem bark of Equation 1 (Table 2),

$a_3$, $a_4$, $a_5$ = respectively, $b_0$, $b_1$, $b_2$ of top wood plus bark of Equation 3 (Table 7), divided by 100.0,

k = percentage of stump wood plus bark at 30 cm stump height (Table 9), and

q = percentages of different stump heights (Table 10).

If the stump (wood plus bark) portion of the stem is also removed from the forest, then the term k·q will not be included in Equation 5. Also, if the merchantable section of stem is expressed in terms of merchantable height, the variable dm/d of Equation 5 will be replaced by hm/h, and the parameters $a_3$, $a_4$, and $a_5$ will be obtained from Table 8.

As an example, let us look at the same white birch tree used in example 2(a) with d = 24.0 cm, h = 20.00 m, stump height = 20.0 cm, and dm = 20.3 cm. If only the net merchantable portion of the stem (wood and bark) is removed from the forest, the logging residues will yield 67.9 kg of ovendry mass. Consequently, the stem section removed will amount to 270.4 - 67.9 = 202.5 kg ovendry mass.

## SUMMARY AND CONCLUSIONS

Nineteen of the most important hardwood species of Ontario were studied in order to estimate aboveground biomass of standing living trees. The following conclusions were drawn, based on the analyses:

1. As in previous studies, predicting ovendry mass of the main tree components and of the whole tree from $d^2h$ can be made more accurately for stem wood and for the whole tree than for the other components. The coefficients of determination are about 0.970 and the standard error of estimate as a percent of the mean about 20% for these better estimations. Somewhat less

accurate but still acceptable results on the foliage and live branches could be due to the social positions of the sample trees in the stand, to various densities of stands in which the sample trees were collected, and to different tree ages. Introducing these into the model as independent variables may improve estimations; however this may cause difficulties in running the biomass inventories.

The examination of bias on the results of these equations, using the whole tree mass equation of the combined species as an example indicated that they can be used with confidence.

It is interesting to observe that, for a given tree size, red oak contains the highest ovendry mass of whole tree, and basswood the lowest. When stem wood is compared, it is seen that sugar maple is the heaviest (red oak, American beech, white elm, and hickory are comparable) and basswood the lightest. For all species combined, based on the equation coefficients given in Table 2, stem wood consists of 64.8%, stem bark 8.5%, live branches 24.9%, and twigs plus leaves 1.8% of the total tree ovendry mass. In the softwoods these were 70.6%, 9.5%, 14.4%, and 5.5%, respectively (Alemdag 1983).

2. Predicting merchantable and unmerchantable portions of the stem as percents of the ovendry mass of the total stem can be done very accurately for all components as evidenced by their $R^2$ and SEE% values. Better results can be obtained for merchantable stem wood and for top wood plus bark than for merchantable stem bark, with the coefficient of multiple determination around 0.800 for the dm/d equations and 0.950 for the hm/h equations. For a given merchantable top diameter within a species, percents of each of these three components change with the change of the breast height diameter. A similar situation is also true when dealing with the merchantable height. After these percentages are calculated for a given tree species for a dm/d or hm/h ratio, they have to be used with the ovendry mass of stem wood plus bark of the tree. Use of these equations requires a measured or estimated ovendry mass of stem wood plus bark. Calculating the percentage values of merchantable and unmerchantable portions of the stem by Equations 3 and 4 is permissible only for the dm/d and hm/h ratios provided in Table 12.

3. If dimensional single-tree data are not available, all of the above equations can be used with stand averages (or with the averages of diameter classes) of the required independent variables.

4. All of these biomass prediction equations can be used with the same degree of reliability for the same tree species growing outside Ontario but under the same ecological conditions.

## ACKNOWLEDGMENTS

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# APPENDIX A

## Tree species studied

## TREE SPECIES STUDIED

| | |
|---|---|
| Trembling aspen | *Populus tremuloides* Michx. |
| Largetooth aspen | *Populus grandidentata* Michx. |
| Balsam poplar | *Populus balsamifera* L. |
| White birch | *Betula papyrifera* Marsh. |
| Yellow birch | *Betula alleghaniensis* Britton |
| Sugar maple | *Acer saccharum* Marsh. |
| Red maple | *Acer rubrum* L. |
| Silver maple | *Acer saccharinum* L. |
| White ash | *Fraxinus americana* L. |
| Black ash | *Fraxinus nigra* Marsh. |
| Red ash | *Fraxinus pennsylvanica* Marsh. |
| Basswood | *Tilia americana* L. |
| American beech | *Fagus grandifolia* Ehrh. |
| Black cherry | *Prunus serotina* Ehrh. |
| White elm | *Ulmus americana* L. |
| Hickory | *Carya* Nutt. spp. |
| Ironwood | *Ostrya virginiana* (Mill.) K. Koch |
| White oak | *Quercus alba* L. |
| Red oak | *Quercus rubra* L. |

# APPENDIX B

## Field, laboratory, and compilation procedures

**Field procedures**

For the biomass and volume measurements, sample trees were randomly selected among living trees. They covered the full diameter distribution of each species, and the height range within 5 cm diameter classes. In selecting these trees, particular attention was paid to those with average vigour, unbroken tops, and sound wood. Before and after felling the trees, the following measurements and material were collected from each sample tree of each species:

(1) Outside-bark diameter at breast height of 1.30 m (d), and total tree height (h);

(2) Outside-bark merchantable diameters (dm) at three locations on the stem: 1/3, 2/3, and top of the height at which a diameter of 9.1 cm occurred; and merchantable heights (hm) from ground level to these three locations on the stem ('merchantable stem' is defined as being the part of the stem from ground level to a minimum outside-bark diameter of 9.1 cm with a minimum length of 2.80 m. This portion then is divided into three equal sections for the data collection purposes (Alemdag 1982));

(3) Outside-bark diameter at the bottom of the tree, at stump height of 0.30 m (in a few cases stump height was different), and at 0.80 m;

(4) Outside-bark diameters at 2.00 m of height and at every 2 m interval up the stem;

(5) Double-bark thickness at each place of diameter measurement except where stem disks were removed;

(6) Total tree age (number of annual rings at breast height plus the age of seedlings growing to this height);

(7) Crown length from the base of the first whorl of live branches to the tip of the tree, and the average crown diameter;

(8) Green mass (GM) of the entire aboveground portion of individual trees by weighing the following components of the trees with a d equal to or larger than 5.1 cm:

(a) each 1/3 section of the merchantable portion, and the top (the unmerchantable portion) of the stem (wood and bark together; bottom 1/3 excluding the stump's green mass),

(b) live branches (in two categories of small and large sizes, wood and bark together), twigs plus leaves, fruits, and dead branches;

(9) Green mass of whole tree where d is between 0.1 cm and 5.0 cm, and green mass of seedlings with a height up to 1.30 m (mass measured in the laboratory);

(10) Sample disks including wood and bark:

(a) from the stem, at breast height, and at the bottom of the middle and the upper merchantable sections and at the bottom of the top (on unmerchantable-size trees, from the breast height and the middle of the stem only),

(b) from the live branches (one from each size category),
(c) from the dead branches;

(11) Sample material from twigs plus leaves, and fruits.

All data were recorded on Fortran coding forms. All lengths and heights were measured and recorded in metres by 0.01 m, all diameters and bark thicknesses in centimetres by 0.1 cm (except crown diameter which is in metres), all masses in kilograms by 0.1 kg.

**Laboratory procedures**
In the laboratory, using samples collected in the field, the following work was performed:

(1) On stem disks, inside-bark diameter and double-bark thickness were measured;

(2) A wedge-shaped piece of wood was cut from each of the stem disks;

(3) In the remaining part of the stem disks and on branch disks, wood and bark were separated and the GM of each was measured by weighing;

(4) The disk wood and disk bark were ovendried and the ovendry mass (OM) of each was measured by weighing;

(5) After removing bark from the wedge, the wedge wood was soaked in water for some time, and its green volume determined by the immersion method; afterwards, this wedge wood was ovendried and its OM measured by weighing;

(6) Twigs and leaves together and fruits were weighed before and after ovendrying.

The samples were dried in a forced-air oven at 105° ± 3°C for 24 to 48 hours or until no change in mass was noted. Samples were weighed to the nearest 0.1 g for their GM and OM values. Wedge-wood volume was measured by 0.1 $cm^3$.

**Compilation procedures**
Mass calculations of the tree components, and wood density and stem volume calculations were conducted using the following procedures:

*Mass calculations.* First, bark percent in terms of wood plus bark of the stem disks were calculated using green mass obtained from the disks. Then, employing the weighted average of these percentages of the two ends of each of the bottom, middle, and upper third sections of the merchantable stem, each of these section's observed green mass of wood plus bark was separated into wood and bark. Weighting factors were the squares of the disks' outside-bark diameters. In the case of the bottom section, disk at breast height was used as the lower-end disk, and in case of the tree top, only one disk was employed. Following this, OM/GM ratios of the above mentioned sample materials were calculated. These ratios were then multiplied by the actual measured GM values of components to arrive at the OM values. When dealing with the wood mass and bark mass of the four stem sections, a weighted average of OM/GM ratios of each section was calculated similarly to the weighted bark

percentages, before applying these ratios to the sections' green masses. Ovendry mass of stump wood and of stump bark were calculated by using the ratio of stump volume to the volume of the part between stump height and the top height of the lower merchantable section. After these calculations for the stem were completed, they were put together to arrive at the ovendry mass of wood and of bark of the total stem. Then, ovendry mass of live branches, twigs plus leaves, fruits, and dead branches were added to this stem total to obtain the ovendry mass of the whole tree. However, when doing estimation analyses, the mass of fruits and dead branches were not included in the whole tree mass. In addition to the ovendry mass of the various tree components, the ovendry mass of the total merchantable stem wood, total merchantable stem bark, and the harvesting residue (whole tree minus merchantable stem wood and bark) were calculated.

*Basic wood density calculations.* The basic wood density by definition is the ratio of ovendry mass of wood to its green volume, expressed in terms of mass per unit of volume. For each disk location on the stem it was calculated by dividing the wedge wood's ovendry mass in grams to its green volume in cubic centimetres. The average wood density of the bole was computed by taking the weighted averages of these wood densities, the weighting factors being the square of the inside-bark diameter of the disks.

*Volume calculations.* Stem volume, from ground level to the top of the tree, was calculated for inside bark and outside bark, in cubic metres. In these calculations the formula for a neiloid frustum was used for the stump volume, the cone formula for the tree top, and Smalian's formula for the part of the stem in between these two sections. The calculated values were presented for the lower third (excluding stump), middle third, and upper third of the merchantable stem, for the top, and for the stump.

The results of these calculations and most of the sample tree information were then entered into computer-produced tables called single-tree summaries. A copy of such a table for a sample tree is given in Appendix C. Subsequently, these processed data were visually checked to see if there were any anomalies among the calculated values, by tabulating them in an ascending order of d and h. The obvious errors were then either corrected by referring to the field data, or the trees with these errors were rejected. The examples of these computer checking tables are also provided in Appendix C. Computer programs to produce these tables and to store the data for further analyses were written in FORTRAN-77/RSX and made operational on the DEC PDP 11/44 with the RSX-11M+ operating system. They were named as ISA42 (Single-Tree Summaries), ISA43 (Checking Table No. 1) and ISA44 (Checking Table No. 2). Copies of these programs can be obtained by writing to the Director, Petawawa National Forestry Institute, Canadian Forestry Service, Agriculture Canada, Chalk River, Ontario, K0J 1J0, Canada.

# APPENDIX C

Computer printouts of single-tree

summary and checking tables

## TABLE OF SINGLE-TREE SUMMARIES

PROJECT NO.: PI-12-067 STUDY NO.: ENFOR-234

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| PLOT NO. 1 | TREE NO. 12 | SPECIES CODE 720 | DBHOB (CM) 35.0 | HEIGHT (M) 22.06 |
| | AGE (YR) 94 | CR DIAM (M) 8.5 | HT BLCR (M) 8.17 | MERCH HT (M) 15.60 |
| | TREE STATUS 1 | DBT AT BH (CM) 2.0 | | |

| ROW NO. | DESCRIPTION | SEC NO. | GREEN MASS (KG) | OVENDRY MASS (KG) | OM/GM RATIO | VOLUME (M***3) OUTSIDE BK | VOLUME (M***3) INSIDE BK |
|---|---|---|---|---|---|---|---|
| 1 | STEM WOOD | 1* | 342.468 | 186.529 | 0.545 | | 0.436 |
| 2 | | 2 | 236.462 | 127.487 | 0.539 | | 0.227 |
| 3 | | 3 | 90.256 | 48.342 | 0.536 | | 0.089 |
| 4 | | 4 | 11.650 | 6.147 | 0.528 | | 0.013 |
| 5 | ROWS 1-4 | TOTAL | 680.836 | 368.506 | 0.541 | | |
| 6 | STEM BARK | 1* | 38.232 | 21.638 | 0.566 | | |
| 7 | | 2 | 27.338 | 15.659 | 0.573 | | |
| 8 | | 3 | 13.044 | 7.305 | 0.560 | | |
| 9 | | 4 | 2.450 | 1.344 | 0.549 | | |
| 10 | ROWS 6-9 | TOTAL | 81.064 | 45.946 | 0.567 | | |
| 11 | STEM WOOD PLUS BARK | 1* | 380.700 | 208.168 | 0.547 | 0.483 | |
| 12 | | 2 | 263.800 | 143.146 | 0.543 | 0.257 | |
| 13 | | 3 | 103.300 | 55.647 | 0.539 | 0.104 | |
| 14 | | 4 | 14.100 | 7.491 | 0.531 | 0.016 | |
| 15 | ROWS 11-14 | TOTAL | 761.900 | 414.452 | 0.544 | | |
| 16 | BARK % OF WOOD PLUS BARK ** | 1 | 10.0 | 10.4 | | | |
| 17 | | 2 | 10.4 | 10.9 | | | |
| 18 | | 3 | 12.6 | 13.1 | | | |
| 19 | | 4 | 17.4 | 17.9 | | | |
| 20 | | AVG. | 10.6 | 11.1 | | | |
| 21 | BRANCHES, DEAD | | 10.100 | 7.070 | 0.700 | | |
| 38 | WOOD, LIVE BRANCHES > 9.0 CM | | 106.049 | 58.962 | 0.556 | | |
| 39 | BARK, LIVE BRANCHES > 9.0 CM | | 20.751 | 11.081 | 0.534 | | |
| 22 | BRANCHES, LIVE > 9.0 CM | | 126.800 | 70.043 | 0.552 | | |
| 40 | WOOD, LIVE BRANCHES < 9.1 CM | | 150.632 | 78.704 | 0.522 | | |
| 41 | BARK, LIVE BRANCHES < 9.1 CM | | 31.768 | 16.661 | 0.524 | | |
| 23 | BRANCHES, LIVE < 9.1 CM | | 182.400 | 95.365 | 0.523 | | |
| 42 | TWIGS | | 16.619 | 8.837 | 0.532 | | |
| 43 | LEAVES | | 20.181 | 7.266 | 0.360 | | |
| 24 | TWIGS AND LEAVES | | 36.800 | 16.103 | 0.438 | | |
| 44 | NEW CONES | | 1.700 | 0.551 | 0.324 | | |
| 45 | OLD CONES | | 0.400 | 0.181 | 0.453 | | |
| 46 | CONES | | 2.100 | 0.732 | 0.349 | | |
| 25 | STUMP WOOD | | 91.078 | 49.892 | 0.548 | | 0.116 |
| 26 | STUMP BARK | | 7.165 | 3.999 | 0.558 | | |
| 27 | STUMP WOOD PLUS BARK | | 98.242 | 53.892 | 0.549 | 0.125 | |
| 28 | TOTAL (ROWS 15,21,22,23,24,27,46) | | 1218.343 | 657.657 | 0.540 | 0.984 | 0.881 |
| 29 | MERCHANTABLE-STEM WOOD | | 669.186 | 362.359 | 0.541 | | 0.752 |
| 30 | MERCHANTABLE-STEM BARK | | 78.614 | 44.602 | 0.567 | | |
| 31 | MERCHANTABLE-STEM WOOD PLUS BARK | | 747.800 | 406.961 | 0.544 | 0.843 | |
| 32 | HARVESTING RESIDUE (28 MINUS 31) | | 468.443 | 249.964 | 0.534 | | |
| 33 | WOOD DENSITY (GRAM/CM**3) | 1 | | | 0.633 | | |
| 34 | | 2 | | | 0.620 | | |
| 35 | | 3 | | | 0.608 | | |
| 36 | | 4 | | | 0.601 | | |
| 37 | | AVG.*** | | | 0.624 | | |

* EXCLUDING STUMP
** WEIGHTED AVERAGE BY DOB**2 OF DISKS
*** WEIGHTED AVERAGE BY DIB**2 OF DISKS

PROJECT: PI-12-067 STUDY: ENFOR-234

CHECKING TABLE NO. 1
NUMBER OF TREES: 5
SPECIES CODE : 720
TREE STATUS : 1

* TOTAL OF THE COLUMN / N
** INCLUDING STUMP
*** DISREGARDING COMPONENTS WITH ZERO VALUES
**** WHOLE TREE CONTAINS DEAD BRANCHES AND CONES

| | | | | OM OF COMPONENTS | | | | | | OM OF COMPONENT/ OM OF STEM WOOD*** | | | | | | | | S W'S |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| DBHOB | TOTAL HEIGHT | PLT NO. | TR NO. | STEM WOOD** | STEM BARK** | DEAD BRCH | LIVE BRCH | TWG,LVS & CONES | WHOLE TREE | MERCH WOOD | MERCH BARK | STUMP W & BK | DEAD BRCH | LIVE BRCH | TWG,LV &CONES | STEM BARK** | WHOLE TREE | OM** /GM** |
| 1.7 | 3.21 | 8 | 3 | 0.19 | 0.02 | 0.00 | 0.23 | 0.12 | 0.56 | 0.00 | 0.00 | 0.28 | 0.00 | 1.21 | 0.66 | 0.10 | 2.98 | 0.63 |
| 6.2 | 5.00 | 1 | 39 | 6.46 | 1.34 | 0.00 | 1.42 | 1.42 | 10.64 | 0.00 | 0.00 | 0.12 | 0.00 | 0.22 | 0.22 | 0.21 | 1.65 | 0.59 |
| 16.1 | 16.30 | 2 | 6 | 60.22 | 9.03 | 1.93 | 14.72 | 2.97 | 88.86 | 0.75 | 0.11 | 0.06 | 0.03 | 0.24 | 0.05 | 0.15 | 1.48 | 0.56 |
| 35.0 | 22.06 | 1 | 12 | 418.40 | 49.94 | 7.07 | 165.41 | 16.84 | 657.66 | 0.87 | 0.11 | 0.13 | 0.02 | 0.40 | 0.04 | 0.12 | 1.57 | 0.54 |
| 59.6 | 23.87 | 8 | 28 | 1085.32 | 201.48 | 3.69 | 649.89 | 13.75 | 1954.13 | 0.93 | 0.18 | 0.07 | 0.00 | 0.60 | 0.01 | 0.19 | 1.80 | 0.60 |
| AVG* | | | | 314.12 | 52.36 | 2.54 | 166.33 | 7.02 | 542.37 | 0.51 | 0.08 | 0.13 | 0.01 | 0.53 | 0.20 | 0.15 | 1.89 | 0.58 |

PROJECT: PI-12-067 STUDY: ENFOR-234

CHECKING TABLE NO. 2
NUMBER OF TREES: 5
SPECIES CODE : 720
TREE STATUS : 1

* TOTAL OF THE COLUMN / N
** INCLUDING STUMP
*** DISREGARDING COMPONENTS WITH ZERO VALUES
**** WHOLE TREE CONTAINS DEAD BRANCHES AND CONES

| | | | | INSIDE VOL. OF 4 SECT'S | | | | INSIDE VOLUME STEM | OM OF WOOD OF 4 SECTIONS | | | | INSIDE OM STEM | WOOD DENSITY OF 4 SECTIONS | | | | | SW OM**/ S INSIDE |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| DBHOB | TOTAL HEIGHT | PLT NO. | TR NO. | 1ST** | 2 ND | 3 RD | 4 TH | WOOD | 1 ST** | 2 ND | 3 RD | 4 TH | WOOD | 1 ST | 2 ND | 3 RD | 4 TH | AVG. | VOLUME** |
| 1.7 | 3.21 | 8 | 3 | 0.000 | 0.000 | 0.000 | 0.000 | 0.001 | 0.000 | 0.000 | 0.000 | 0.000 | 0.188 | 0.649 | 0.687 | 0.000 | 0.000 | 0.664 | 188.0 |
| 6.2 | 5.00 | 1 | 39 | 0.000 | 0.000 | 0.000 | 0.000 | 0.008 | 0.000 | 0.000 | 0.000 | 0.000 | 6.457 | 0.602 | 0.517 | 0.000 | 0.000 | 0.574 | 807.1 |
| 16.1 | 16.30 | 2 | 6 | 0.041 | 0.026 | 0.019 | 0.018 | 0.104 | 20.678 | 15.414 | 12.309 | 11.816 | 60.217 | 0.602 | 0.527 | 0.583 | 0.536 | 0.569 | 579.0 |
| 35.0 | 22.06 | 1 | 12 | 0.552 | 0.227 | 0.089 | 0.013 | 0.881 | 236.421 | 127.487 | 48.342 | 6.147 | 418.398 | 0.633 | 0.620 | 0.608 | 0.601 | 0.624 | 474.9 |
| 59.6 | 23.87 | 8 | 28 | 1.543 | 0.506 | 0.134 | 0.005 | 2.189 | 656.491 | 321.168 | 103.205 | 4.455 | 1085.320 | 0.676 | 0.720 | 0.628 | 0.596 | 0.683 | 495.8 |
| AVG* | | | | 0.427 | 0.152 | 0.048 | 0.007 | 0.637 | 182.718 | 92.814 | 32.771 | 4.484 | 314.116 | 0.632 | 0.614 | 0.364 | 0.347 | 0.623 | 509.0 |

## APPENDIX D

Tables for the main tree components: 1-5

**Table 1. Statistical data for trees of component-mass analysis**

| Species | Number of sample trees n | d(cm) Mean | d(cm) SD+ | d(cm) Range | h(m) Mean | h(m) SD+ | h(m) Range | GM of whole tree (kg) Mean | GM of whole tree (kg) SD+ | GM of whole tree (kg) Range |
|---|---|---|---|---|---|---|---|---|---|---|
| Trembling aspen | 224 | 16.6 | 7.7 | 5.2 - 43.5 | 17.4 | 4.8 | 7.0 - 27.3 | 257.5 | 298.3 | 8.7 - 1854.9 |
| Largetooth aspen | 96 | 16.3 | 7.1 | 5.4 - 39.2 | 17.5 | 5.0 | 7.5 - 28.9 | 216.6 | 222.7 | 8.9 - 1195.0 |
| Balsam poplar | 90 | 24.9 | 9.9 | 6.6 - 53.2 | 18.5 | 4.9 | 6.2 - 27.0 | 603.6 | 613.7 | 19.9 - 3091.9 |
| White birch | 135 | 17.0 | 6.1 | 5.4 - 32.7 | 16.5 | 4.0 | 7.3 - 22.3 | 246.5 | 191.9 | 12.8 - 1162.4 |
| Yellow birch | 95 | 35.4 | 16.7 | 6.2 - 70.3 | 19.7 | 4.8 | 5.0 - 25.6 | 1495.6 | 1276.5 | 19.4 - 4971.7 |
| Sugar maple | 112 | 27.1 | 14.9 | 5.1 - 57.8 | 18.1 | 4.9 | 7.3 - 26.4 | 925.5 | 957.1 | 9.5 - 3928.4 |
| Red maple | 63 | 21.4 | 10.7 | 5.7 - 45.2 | 16.7 | 5.0 | 8.0 - 25.4 | 437.6 | 467.3 | 17.1 - 1690.0 |
| Silver maple | 37 | 24.3 | 12.1 | 5.3 - 45.3 | 20.4 | 5.1 | 9.5 - 26.4 | 620.7 | 605.2 | 19.6 - 1953.7 |
| White ash | 74 | 23.8 | 11.4 | 5.9 - 53.7 | 18.0 | 3.6 | 9.1 - 26.9 | 502.7 | 464.0 | 15.8 - 2738.1 |
| Black ash | 26 | 15.0 | 7.6 | 5.7 - 33.1 | 13.7 | 3.2 | 7.2 - 20.3 | 175.6 | 177.9 | 14.0 - 597.9 |
| Red ash§ | 28 | 21.2 | 8.8 | 6.9 - 40.2 | 18.3 | 5.1 | 7.2 - 26.7 | 377.3 | 303.0 | 18.3 - 1001.8 |
| Basswood | 76 | 27.7 | 13.7 | 5.3 - 54.8 | 18.1 | 5.5 | 3.8 - 26.1 | 665.6 | 631.8 | 4.5 - 2201.7 |
| American beech | 76 | 26.5 | 10.9 | 6.1 - 46.3 | 19.1 | 5.0 | 5.6 - 26.5 | 782.9 | 657.5 | 14.6 - 2478.7 |
| Black cherry | 72 | 24.1 | 10.7 | 6.4 - 49.6 | 17.8 | 4.1 | 8.4 - 25.9 | 514.9 | 433.4 | 20.2 - 1989.4 |
| White elm | 77 | 21.2 | 9.3 | 5.4 - 55.2 | 14.0 | 3.5 | 6.6 - 23.2 | 378.3 | 455.9 | 8.3 - 3080.6 |
| Hickory | 73 | 22.0 | 9.7 | 5.5 - 46.6 | 20.3 | 6.0 | 7.9 - 29.4 | 659.3 | 629.6 | 9.6 - 2390.8 |
| Ironwood | 14 | 7.7 | 3.5 | 5.2 - 18.5 | 8.6 | 1.7 | 6.3 - 11.9 | 30.1 | 34.3 | 9.7 - 131.9 |
| White oak | 61 | 24.6 | 15.9 | 5.5 - 74.3 | 11.9 | 4.5 | 5.0 - 21.5 | 573.1 | 858.8 | 10.4 - 3988.6 |
| Red oak | 114 | 23.8 | 10.0 | 5.5 - 55.3 | 16.0 | 3.7 | 6.5 - 23.0 | 575.1 | 551.9 | 11.6 - 3421.6 |
| All hardwoods | 1543 | 22.4 | 12.0 | 5.1 - 74.3 | 17.3 | 5.0 | 3.8 - 29.4 | 544.4 | 684.9 | 4.5 - 4971.7 |

*GM of whole tree is green mass of tree above ground including stem, live branches, twigs and leaves.
+SD is standard deviation.
§Red ash includes some sample trees from green ash (*Fraxinus pennsylvanica* var. *subintegerrima* [Vahl] Fern.).

**Table 2. Regression coefficients and statistics of Equation 1: $OM = b_1 \cdot d^2h$**

| Component | $b_1$ | $r^2$ | SEE% | Mean (kg) | Range (kg) |
|---|---|---|---|---|---|
| | | Trembling aspen (n = 224) | | | |
| Stem wood | 0.014579 | 0.974 | 18.4 | 102.9 | 3.2 - 671.6 |
| Stem bark | 0.003198 | 0.918 | 36.7 | 21.4 | 0.6 - 133.1 |
| Live branches | 0.002498 | 0.792 | 72.7 | 15.2 | 0.3 - 174.8 |
| Twigs plus leaves | 0.000510 | 0.619 | 63.2 | 4.2 | 0.2 - 23.2 |
| Whole tree | 0.020785 | 0.972 | 19.7 | 143.7 | 5.1 - 964.4 |
| Dead branches* | - | - | - | - | - |
| | | Largetooth aspen (n = 96) | | | |
| Stem wood | 0.013427 | 0.981 | 13.9 | 86.7 | 2.9 - 403.4 |
| Stem bark | 0.002931 | 0.917 | 28.7 | 19.4 | 1.0 - 112.4 |
| Live branches | 0.001840 | 0.714 | 76.9 | 10.7 | 0.3 - 116.9 |
| Twigs plus leaves | 0.000379 | 0.736 | 54.0 | 2.6 | 0.2 - 20.7 |
| Whole tree | 0.018577 | 0.982 | 13.7 | 119.4 | 4.6 - 641.2 |
| Dead branches* | - | - | - | - | - |
| | | Balsam poplar (n = 90) | | | |
| Stem wood | 0.013164 | 0.978 | 15.3 | 202.1 | 4.5 - 996.3 |
| Stem bark | 0.001888 | 0.908 | 30.7 | 29.9 | 1.3 - 164.2 |
| Live branches | 0.003150 | 0.781 | 58.2 | 46.7 | 2.0 - 352.6 |
| Twigs plus leaves | 0.000303 | 0.491 | 51.8 | 5.9 | 0.5 - 20.2 |
| Whole tree | 0.018505 | 0.974 | 16.9 | 284.6 | 9.6 - 1516.3 |
| Dead branches* | 0.000677 | 0.491 | 143.8 | 8.6 | 0.0 - 124.7 |
| | | White birch (n = 135) | | | |
| Stem wood | 0.016211 | 0.976 | 11.3 | 99.7 | 3.4 - 350.8 |
| Stem bark | 0.002873 | 0.785 | 37.7 | 17.7 | 0.7 - 89.9 |
| Live branches | 0.003525 | 0.588 | 85.1 | 18.8 | 0.7 - 222.9 |
| Twigs plus leaves | 0.000859 | 0.745 | 43.1 | 5.2 | 0.4 - 25.6 |
| Whole tree | 0.023468 | 0.967 | 14.1 | 141.4 | 6.2 - 657.1 |
| Dead branches* | - | - | - | - | - |

*Ovendry mass of dead branches is not included in the whole tree ovendry mass.

Table 2. (cont'd)

| Component | $b_1$ | $r^2$ | SEE% | Mean (kg) | Range (kg) |
|---|---|---|---|---|---|
| | | Yellow birch (n = 95) | | | |
| Stem wood | 0.015339 | 0.932 | 21.2 | 546.5 | 6.5 - 1623.9 |
| Stem bark | 0.002193 | 0.773 | 43.0 | 77.7 | 1.3 279.0 |
| Live branches | 0.006947 | 0.774 | 52.0 | 226.5 | 1.3 - 1110.0 |
| Twigs plus leaves | 0.000343 | 0.430 | 68.1 | 13.3 | 1.0 - 68.3 |
| Whole tree | 0.024822 | 0.927 | 23.3 | 864.1 | 10.6 - 2951.4 |
| Dead branches* | 0.000001 | 0.424 | 138.3 | 15.4 | 0.0 - 164.8 |
| | | Sugar maple (n = 112) | | | |
| Stem wood | 0.017806 | 0.983 | 13.5 | 371.1 | 3.8 - 1536.4 |
| Stem bark | 0.001675 | 0.714 | 48.7 | 39.6 | 0.5 - 233.7 |
| Live branches | 0.006717 | 0.890 | 37.7 | 136.8 | 0.2 - 684.5 |
| Twigs plus leaves | 0.000526 | 0.655 | 52.8 | 12.7 | 0.2 - 49.6 |
| Whole tree | 0.026724 | 0.978 | 15.2 | 560.1 | 6.2 - 2421.1 |
| Dead branches* | 0.000297 | 0.381 | 120.6 | 6.4 | 0.0 - 46.3 |
| | | Red maple (n = 63) | | | |
| Stem wood | 0.014497 | 0.962 | 20.7 | 173.2 | 5.6 - 691.1 |
| Stem bark | 0.001562 | 0.857 | 34.6 | 20.6 | 0.8 - 59.8 |
| Live branches | 0.005162 | 0.814 | 55.6 | 58.1 | 1.6 - 347.6 |
| Twigs plus leaves | 0.000499 | 0.713 | 45.7 | 7.0 | 0.4 - 26.3 |
| Whole tree | 0.021720 | 0.970 | 18.6 | 258.9 | 10.2 - 972.8 |
| Dead branches* | 0.000506 | 0.311 | 140.3 | 6.4 | 0.0 - 52.0 |
| | | Silver maple (n = 37) | | | |
| Stem wood | 0.013607 | 0.973 | 14.4 | 245.5 | 6.4 - 722.2 |
| Stem bark | 0.001326 | 0.956 | 18.7 | 23.9 | 0.8 - 73.2 |
| Live branches | 0.006684 | 0.803 | 59.0 | 102.6 | 0.7 - 492.1 |
| Twigs plus leaves | 0.000394 | 0.707 | 45.5 | 7.7 | 0.7 - 26.8 |
| Whole tree | 0.022011 | 0.968 | 17.2 | 379.7 | 11.6 - 1163.7 |
| Dead branches* | 0.000169 | 0.206 | 164.6 | 3.1 | 0.0 - 26.5 |

*Ovendry mass of dead branches is not included in the whole tree ovendry mass.

Table 2. (cont'd)

| Component | $b_1$ | $r^2$ | SEE% | Mean (kg) | Range (kg) |
|---|---|---|---|---|---|
| | | White ash (n = 74) | | | |
| Stem wood | 0.015349 | 0.940 | 21.8 | 215.4 | 8.2 - 1077.8 |
| Stem bark | 0.001810 | 0.814 | 32.5 | 27.8 | 1.1 - 103.7 |
| Live branches | 0.006197 | 0.739 | 69.3 | 75.1 | 0.6 - 653.4 |
| Twigs plus leaves | 0.000328 | 0.186 | 64.0 | 5.8 | 0.1 - 15.8 |
| Whole tree | 0.023684 | 0.931 | 24.9 | 324.2 | 0.7 - 1850.1 |
| Dead branches* | 0.000855 | 0.264 | 192.7 | 10.4 | 0.0 - 93.6 |
| | | Black ash (n = 26) | | | |
| Stem wood | 0.015574 | 0.980 | 15.6 | 71.3 | 5.3 - 286.8 |
| Stem bark | 0.001914 | 0.720 | 43.4 | 10.5 | 1.0 - 32.0 |
| Live branches | 0.001808 | 0.405 | 98.3 | 9.4 | 0.6 - 50.2 |
| Twigs plus leaves | 0.000315 | 0.307 | 76.3 | 1.9 | 0.2 - 6.7 |
| Whole tree | 0.019611 | 0.957 | 21.3 | 93.2 | 7.9 - 326.2 |
| Dead branches* | 0.001724 | 0.780 | 87.7 | 6.0 | 0.0 - 43.2 |
| | | Red ash (n = 28) | | | |
| Stem wood | 0.012970 | 0.932 | 21.8 | 149.1 | 5.9 - 418.2 |
| Stem bark | 0.002161 | 0.880 | 27.0 | 25.7 | 1.4 - 73.6 |
| Live branches | 0.002593 | 0.836 | 38.3 | 29.0 | 0.8 - 97.2 |
| Twigs plus leaves | 0.000326 | 0.268 | 51.9 | 4.6 | 0.1 - 9.1 |
| Whole tree | 0.018050 | 0.942 | 19.8 | 208.4 | 9.9 - 587.8 |
| Dead branches* | 0.000190 | 0.179 | 199.4 | 2.1 | 0.0 - 20.4 |
| | | Basswood (n = 76) | | | |
| Stem wood | 0.011626 | 0.972 | 16.4 | 240.1 | 1.4 - 822.2 |
| Stem bark | 0.001877 | 0.826 | 39.3 | 40.9 | 0.2 - 144.6 |
| Live branches | 0.002677 | 0.844 | 42.6 | 54.5 | 0.4 - 202.5 |
| Twigs plus leaves | 0.000329 | 0.445 | 68.1 | 8.0 | 0.1 - 32.3 |
| Whole tree | 0.016509 | 0.973 | 15.8 | 343.6 | 2.2 - 1118.9 |
| Dead branches* | 0.000131 | 0.420 | 108.7 | 2.8 | 0.0 - 17.0 |

*Ovendry mass of dead branches is not included in the whole tree ovendry mass.

Table 2. (cont'd)

| Component | $b_1$ | $r^2$ | SEE% | Mean (kg) | Range (kg) |
|---|---|---|---|---|---|
| | | American beech (n = 76) | | | |
| Stem wood | 0.017437 | 0.897 | 25.3 | 317.7 | 5.5 - 1013.8 |
| Stem bark | 0.001230 | 0.754 | 36.5 | 23.5 | 0.9 - 74.5 |
| Live branches | 0.005880 | 0.647 | 64.7 | 100.6 | 0.7 - 444.1 |
| Twigs plus leaves | 0.000356 | 0.315 | 80.9 | 7.1 | 0.4 - 31.2 |
| Whole tree | 0.024903 | 0.878 | 28.6 | 449.0 | 8.3 - 1372.5 |
| Dead branches* | 0.000545 | 0.326 | 124.4 | 9.4 | 0.0 - 55.4 |
| | | Black cherry (n = 72) | | | |
| Stem wood | 0.014529 | 0.826 | 31.7 | 212.0 | 8.7 - 689.6 |
| Stem bark | 0.001741 | 0.666 | 43.2 | 26.4 | 1.3 - 82.8 |
| Live branches | 0.005579 | 0.719 | 71.4 | 65.3 | 0.4 - 403.7 |
| Twigs plus leaves | 0.000275 | 0.269 | 68.2 | 4.5 | 0.2 - 15.0 |
| Whole tree | 0.022124 | 0.901 | 26.1 | 308.2 | 12.1 - 1183.5 |
| Dead branches* | 0.001129 | 0.322 | 105.7 | 16.2 | 0.0 - 95.4 |
| | | White elm (n = 77) | | | |
| Stem wood | 0.017416 | 0.970 | 23.0 | 146.7 | 2.8 - 1328.9 |
| Stem bark | 0.002443 | 0.803 | 68.9 | 19.7 | 0.5 - 246.7 |
| Live branches | 0.003957 | 0.647 | 58.7 | 42.7 | 0.7 - 234.8 |
| Twigs plus leaves | 0.000404 | 0.517 | 53.4 | 4.9 | 0.3 - 22.0 |
| Whole tree | 0.024220 | 0.962 | 23.7 | 213.9 | 4.3 - 1832.2 |
| Dead branches* | 0.000067 | 0.029 | 175.3 | 0.9 | 0.0 - 7.0 |
| | | Hickory (n = 73) | | | |
| Stem wood | 0.017007 | 0.940 | 21.4 | 241.1 | 3.7 - 857.6 |
| Stem bark | 0.002114 | 0.899 | 26.2 | 30.8 | 0.8 - 121.2 |
| Live branches | 0.008546 | 0.769 | 61.1 | 106.0 | 0.9 - 621.4 |
| Twigs plus leaves | 0.000667 | 0.813 | 41.8 | 9.3 | 0.3 - 37.0 |
| Whole tree | 0.028334 | 0.949 | 21.1 | 387.3 | 5.7 - 1335.1 |
| Dead branches* | 0.000106 | 0.026 | 256.6 | 1.8 | 0.0 - 31.0 |

*Ovendry mass of dead branches is not included in the whole tree ovendry mass.

Table 2. (cont'd)

| Component | $b_1$ | $r^2$ | SEE% | Mean (kg) | Range (kg) |
|---|---|---|---|---|---|
| | | Ironwood (n = 14) | | | |
| Stem wood | 0.015409 | 0.947 | 23.8 | 12.1 | 3.0 - 48.1 |
| Stem bark | 0.001432 | 0.593 | 45.2 | 1.4 | 0.6 - 3.9 |
| Live branches | 0.004147 | 0.953 | 29.8 | 2.8 | 0.3 - 13.7 |
| Twigs plus leaves | 0.002432 | 0.906 | 35.6 | 1.8 | 0.7 - 8.9 |
| Whole tree | 0.023420 | 0.963 | 20.4 | 18.1 | 6.3 - 74.5 |
| Dead branches* | - | - | - | - | - |
| | | White oak (n = 61) | | | |
| Stem wood | 0.012846 | 0.959 | 27.9 | 200.5 | 2.8 - 1362.8 |
| Stem bark | 0.001608 | 0.866 | 47.9 | 26.9 | 0.7 - 162.5 |
| Live branches | 0.007350 | 0.979 | 26.1 | 95.4 | 0.0 - 843.0 |
| Twigs plus leaves | 0.000460 | 0.898 | 45.2 | 7.2 | 0.0 - 58.8 |
| Whole tree | 0.022264 | 0.978 | 22.1 | 329.9 | 4.7 - 2385.5 |
| Dead branches* | 0.002018 | 0.521 | 220.4 | 2.1 | 0.0 - 406.9 |
| | | Red oak (n = 114) | | | |
| Stem wood | 0.017601 | 0.936 | 21.6 | 207.5 | 5.2 - 887.4 |
| Stem bark | 0.003002 | 0.817 | 34.8 | 37.0 | 0.7 - 156.6 |
| Live branches | 0.008438 | 0.718 | 79.2 | 80.2 | 0.2 - 896.2 |
| Twigs plus leaves | 0.000669 | 0.624 | 51.7 | 8.5 | 0.3 - 37.3 |
| Whole tree | 0.029710 | 0.950 | 21.4 | 333.2 | 6.5 - 1977.5 |
| Dead branches* | 0.001242 | 0.430 | 103.8 | 14.1 | 0.0 - 158.9 |
| | | All hardwoods (n = 1543) | | | |
| Stem wood | 0.015220 | 0.936 | 30.5 | 208.1 | 1.4 - 1623.9 |
| Stem bark | 0.001992 | 0.769 | 55.8 | 29.3 | 0.2 - 279.0 |
| Live branches | 0.005859 | 0.737 | 90.5 | 67.8 | 0.0 - 1110.0 |
| Twigs plus leaves | 0.000415 | 0.555 | 73.1 | 6.7 | 0.0 - 68.3 |
| Whole tree | 0.023486 | 0.928 | 34.0 | 311.9 | 2.2 - 2951.4 |
| Dead branches* | 0.000566 | 0.245 | 243.3 | 7.0 | 0.0 - 406.9 |

*Ovendry mass of dead branches is not included in the whole tree ovendry mass.

**Table 3. Component, whole tree, and dead-branches ovendry mass as percent of stem wood ovendry mass**

| Species | Number of sample trees n | Stem bark | Live branches | Twigs plus leaves | Whole tree | Dead branches* |
|---|---|---|---|---|---|---|
| Trembling aspen | 224 | 21.9 | 17.1 | 3.5 | 142.5 | - |
| Largetooth aspen | 96 | 21.8 | 13.7 | 2.8 | 138.3 | - |
| Balsam poplar | 90 | 14.4 | 23.9 | 2.3 | 140.6 | 5.1 |
| White birch | 135 | 17.7 | 21.7 | 5.3 | 144.7 | - |
| Yellow birch | 95 | 14.3 | 45.3 | 2.2 | 161.8 | 0.0 |
| Sugar maple | 112 | 9.4 | 37.7 | 3.0 | 150.1 | 1.7 |
| Red maple | 63 | 10.8 | 35.6 | 3.4 | 149.8 | 3.5 |
| Silver maple | 37 | 9.8 | 49.1 | 2.9 | 161.8 | 1.2 |
| White ash | 74 | 11.8 | 40.4 | 2.1 | 154.3 | 5.6 |
| Black ash | 26 | 12.3 | 11.6 | 2.0 | 125.9 | 11.1 |
| Red ash | 28 | 16.7 | 20.0 | 2.5 | 139.2 | 1.5 |
| Basswood | 76 | 16.2 | 23.0 | 2.8 | 142.0 | 1.1 |
| American beech | 76 | 7.1 | 33.7 | 2.0 | 142.8 | 3.1 |
| Black cherry | 72 | 12.0 | 38.4 | 1.9 | 152.3 | 7.8 |
| White elm | 77 | 14.0 | 22.7 | 2.3 | 139.0 | 0.4 |
| Hickory | 73 | 12.4 | 50.3 | 3.9 | 166.6 | 0.5 |
| Ironwood | 14 | 9.3 | 26.9 | 15.8 | 152.0 | - |
| White oak | 61 | 12.5 | 57.2 | 3.6 | 173.3 | 15.7 |
| Red oak | 114 | 17.1 | 47.9 | 3.8 | 168.8 | 7.1 |
| All hardwoods | 1543 | 13.1 | 38.5 | 2.7 | 154.3 | 3.7 |

*Ovendry mass of dead branches is not included in the whole tree ovendry mass.

**Table 4. Ovendry mass/green mass ratios**

| Species | Number of sample trees n | Stem wood | Stem bark | Live branches | Twigs plus leaves | Whole tree |
|---|---|---|---|---|---|---|
| Trembling aspen | 224 | 0.573 | 0.580 | 0.510 | 0.372 | 0.558 |
| Largetooth aspen | 96 | 0.560 | 0.576 | 0.530 | 0.333 | 0.551 |
| Balsam poplar | 90 | 0.472 | 0.474 | 0.484 | 0.375 | 0.472 |
| White birch | 135 | 0.573 | 0.663 | 0.575 | 0.400 | 0.574 |
| Yellow birch | 95 | 0.580 | 0.585 | 0.584 | 0.411 | 0.578 |
| Sugar maple | 112 | 0.620 | 0.596 | 0.591 | 0.429 | 0.605 |
| Red maple | 63 | 0.612 | 0.558 | 0.570 | 0.444 | 0.592 |
| Silver maple | 37 | 0.630 | 0.568 | 0.597 | 0.441 | 0.612 |
| White ash | 74 | 0.664 | 0.583 | 0.652 | 0.373 | 0.645 |
| Black ash | 26 | 0.544 | 0.513 | 0.510 | 0.354 | 0.531 |
| Red ash | 28 | 0.568 | 0.542 | 0.523 | 0.377 | 0.552 |
| Basswood | 76 | 0.528 | 0.506 | 0.499 | 0.397 | 0.516 |
| American beech | 76 | 0.585 | 0.558 | 0.553 | 0.442 | 0.573 |
| Black cherry | 72 | 0.616 | 0.556 | 0.584 | 0.388 | 0.599 |
| White elm | 77 | 0.574 | 0.565 | 0.565 | 0.390 | 0.566 |
| Hickory | 73 | 0.609 | 0.512 | 0.593 | 0.380 | 0.587 |
| Ironwood | 14 | 0.632 | 0.569 | 0.603 | 0.465 | 0.601 |
| White oak | 61 | 0.584 | 0.588 | 0.570 | 0.420 | 0.576 |
| Red oak | 114 | 0.568 | 0.655 | 0.597 | 0.442 | 0.579 |
| All hardwoods | 1543 | 0.581 | 0.571 | 0.573 | 0.404 | 0.573 |

**Table 5. Average basic wood densities**

| Species | Basic wood density ($kg/m^3$) | | Number of sample trees | | Number of specimens* |
|---|---|---|---|---|---|
| Trembling aspen | 390 | (374)+ | 54 | (20)+ | 164 |
| Largetooth aspen | 363 | (390) | 19 | (10) | 60 |
| Balsam poplar | 354 | (372) | 87 | (10) | 348 |
| White birch | 536 | (506) | 56 | (16) | 200 |
| Yellow birch | 596 | (559) | 95 | (25) | 368 |
| Sugar maple | 623 | (597) | 112 | (19) | 402 |
| Red maple | 583 | (516) | 63 | (6) | 198 |
| Silver maple | 477 | (461) | 37 | (5) | 136 |
| White ash | 594 | (570) | 64 | (13) | 256 |
| Black ash | 545 | (468) | 18 | (5) | 72 |
| Red ash | 555 | (373) | 24 | (6) | 96 |
| Basswood | 426 | (360) | 76 | (4) | 288 |
| American beech | 610 | (590) | 76 | (17) | 292 |
| Black cherry | 569 | (510) | 64 | (5) | 256 |
| White elm | 580 | (524) | 68 | (23) | 272 |
| Hickory | 616 | (628) | 67 | (5) | 268 |
| Ironwood | 652 | (652) | 14 | (6) | 28 |
| White oak | 646 | (654) | 49 | (5) | 196 |
| Red oak | 590 | - | 100 | - | 400 |
| All hardwoods | 548 | - | 1143 | - | 4300 |

*Number of wedges taken from the disks (one from each) in order to determine basic wood density.

+Figures in parentheses are from Jessome (1977).

# APPENDIX E

Tables for the merchantable and unmerchantable components of the stem: 6-13

**Table 6. Statistical data for trees of merchantable-mass analysis**

| Species | Number of sample trees n | Number of observations | d(cm) Mean | d(cm) Range | h(m) Mean | h(m) Range | dm/d* Mean | dm/d* Range | hm/h* Mean | hm/h* Range |
|---|---|---|---|---|---|---|---|---|---|---|
| Trembling aspen | 164 | 492 | 19.5 | 10.1 - 43.5 | 19.53 | 9.58 - 27.25 | 0.731 | 0.209 - 0.995 | 0.422 | 0.083 - 0.852 |
| Largetooth aspen | 71 | 213 | 19.2 | 9.6 - 39.2 | 19.71 | 11.60 - 28.90 | 0.716 | 0.232 - 0.990 | 0.429 | 0.066 - 0.849 |
| Balsam poplar | 87 | 261 | 25.5 | 10.0 - 53.2 | 18.81 | 8.70 - 27.00 | 0.637 | 0.174 - 0.990 | 0.458 | 0.121 - 0.884 |
| White birch | 103 | 309 | 19.4 | 10.0 - 32.7 | 18.21 | 11.70 - 22.25 | 0.716 | 0.278 - 0.992 | 0.426 | 0.071 - 0.797 |
| Yellow birch | 89 | 267 | 37.3 | 10.4 - 70.3 | 20.47 | 10.00 - 25.60 | 0.598 | 0.136 - 0.991 | 0.468 | 0.088 - 0.874 |
| Sugar maple | 89 | 267 | 31.6 | 10.0 - 57.8 | 19.82 | 9.86 - 26.41 | 0.639 | 0.157 - 1.000 | 0.445 | 0.076 - 0.838 |
| Red maple | 36 | 108 | 28.1 | 13.5 - 45.2 | 20.04 | 10.76 - 25.35 | 0.620 | 0.201 - 0.993 | 0.449 | 0.152 - 0.804 |
| Silver maple | 31 | 93 | 27.4 | 13.3 - 45.3 | 21.99 | 14.15 - 26.38 | 0.606 | 0.205 - 0.933 | 0.441 | 0.106 - 0.812 |
| White ash | 64 | 192 | 26.3 | 10.7 - 53.7 | 18.83 | 11.75 - 26.93 | 0.626 | 0.169 - 0.975 | 0.451 | 0.138 - 0.837 |
| Black ash | 17 | 51 | 18.8 | 10.7 - 33.1 | 15.48 | 12.60 - 20.30 | 0.728 | 0.248 - 0.991 | 0.401 | 0.107 - 0.775 |
| Red ash | 24 | 72 | 23.3 | 12.0 - 40.2 | 19.66 | 13.50 - 26.70 | 0.669 | 0.226 - 0.950 | 0.433 | 0.129 - 0.828 |
| Basswood | 68 | 204 | 30.0 | 11.5 - 54.8 | 19.31 | 9.41 - 26.10 | 0.647 | 0.166 - 0.993 | 0.473 | 0.121 - 0.866 |
| American beech | 70 | 210 | 28.0 | 10.5 - 46.3 | 19.98 | 9.72 - 26.50 | 0.635 | 0.197 - 0.989 | 0.468 | 0.106 - 0.858 |
| Black cherry | 63 | 189 | 26.4 | 10.1 - 49.6 | 18.64 | 8.35 - 25.92 | 0.614 | 0.183 - 0.991 | 0.468 | 0.138 - 0.866 |
| White elm | 67 | 201 | 23.2 | 11.3 - 55.2 | 14.74 | 8.11 - 23.24 | 0.643 | 0.165 - 0.992 | 0.400 | 0.083 - 0.824 |
| Hickory | 67 | 201 | 23.5 | 10.0 - 46.6 | 21.25 | 11.60 - 29.40 | 0.643 | 0.204 - 0.990 | 0.426 | 0.076 - 0.821 |
| White oak | 45 | 135 | 29.9 | 12.2 - 74.3 | 13.48 | 5.00 - 21.50 | 0.609 | 0.125 - 0.994 | 0.434 | 0.149 - 0.834 |
| Red oak | 100 | 300 | 25.6 | 10.1 - 55.3 | 16.64 | 9.92 - 23.00 | 0.629 | 0.180 - 0.998 | 0.452 | 0.106 - 0.889 |
| All hardwoods | 1255 | 3765 | 25.6 | 9.6 - 74.3 | 18.81 | 5.00 - 29.40 | 0.660 | 0.125 - 1.000 | 0.445 | 0.066 - 0.889 |

*dm is merchantable top diameter and hm is merchantable height.

**Table 7. Regression coefficients and statistics of Equation 3: $OM\% = b_0 + b_1 \cdot (dm/d) + b_2 \cdot (dm/d)^2$**

| Component | Regression coefficients $b_0$ | $b_1$ | $b_2$ | $R^2$ | SEE% | Mean (%) | Range (%) |
|---|---|---|---|---|---|---|---|
| | | | Trembling aspen (n = 164) | | | | |
| Merchantable stem wood | 63.449 | 102.391 | -144.963 | 0.894 | 11.18 | 55.4 | 13.7 - 88.0 |
| Merchantable stem bark | 16.920 | 5.385 | -18.227 | 0.660 | 25.24 | 10.5 | 2.6 - 24.7 |
| Top wood plus bark | 19.631 | -107.776 | 163.190 | 0.911 | 19.86 | 34.1 | 0.2 - 83.5 |
| | | | Largetooth aspen (n = 71) | | | | |
| Merchantable stem wood | 67.474 | 82.794 | -131.142 | 0.902 | 10.31 | 55.0 | 15.1 - 86.7 |
| Merchantable stem bark | 14.522 | 20.714 | -31.798 | 0.730 | 20.65 | 12.0 | 3.4 - 22.4 |
| Top wood plus bark | 18.004 | -103.508 | 162.940 | 0.925 | 18.35 | 33.0 | 0.5 - 81.1 |
| | | | Balsam poplar (n = 87) | | | | |
| Merchantable stem wood | 91.781 | 4.388 | -69.670 | 0.788 | 13.3 | 63.4 | 23.0 - 90.1 |
| Merchantable stem bark | 13.225 | 1.641 | -11.409 | 0.483 | 27.9 | 91.7 | 2.5 - 24.2 |
| Top wood plus bark | -5.006 | -6.029 | 81.079 | 0.801 | 34.1 | 27.4 | 0.4 - 71.9 |
| | | | White birch (n = 103) | | | | |
| Merchantable stem wood | 59.476 | 121.358 | -159.218 | 0.901 | 9.94 | 59.2 | 14.4 - 87.2 |
| Merchantable stem bark | 13.736 | 9.510 | -19.791 | 0.551 | 29.91 | 9.7 | 2.4 - 39.8 |
| Top wood plus bark | 26.788 | -130.868 | 179.009 | 0.912 | 20.88 | 31.1 | 1.2 - 83.2 |
| | | | Yellow birch (n = 89) | | | | |
| Merchantable stem wood | 84.625 | 31.697 | -89.097 | 0.805 | 12.6 | 66.2 | 19.6 - 93.0 |
| Merchantable stem bark | 11.620 | 4.975 | -12.759 | 0.313 | 37.3 | 9.3 | 2.0 - 25.3 |
| Top wood plus bark | 3.755 | -36.672 | 101.856 | 0.839 | 34.6 | 24.5 | 0.2 - 74.8 |
| | | | Sugar maple (n = 89) | | | | |
| Merchantable stem wood | 81.135 | 62.534 | -117.722 | 0.869 | 11.2 | 65.7 | 15.6 - 94.9 |
| Merchantable stem bark | 6.479 | 17.474 | -20.842 | 0.376 | 31.4 | 7.9 | 1.8 - 18.2 |
| Top wood plus bark | 12.386 | -80.008 | 138.564 | 0.882 | 28.9 | 26.4 | 0.3 - 81.8 |

Table 7. (cont'd)

| Component | Regression coefficients $b_0$ | $b_1$ | $b_2$ | $R^2$ | SEE% | Mean (%) | Range (%) |
|---|---|---|---|---|---|---|---|
| | | | Red maple (n = 36) | | | | |
| Merchantable stem wood | 83.812 | 42.894 | -98.309 | 0.870 | 9.6 | 67.7 | 31.1 - 92.2 |
| Merchantable stem bark | 7.023 | 15.746 | -19.931 | 0.431 | 26.9 | 8.1 | 3.3 - 15.4 |
| Top wood plus bark | 9.165 | -58.640 | 118.240 | 0.875 | 29.1 | 24.2 | 0.6 - 63.0 |
| | | | Silver maple (n = 31) | | | | |
| Merchantable stem wood | 84.812 | 49.197 | -113.081 | 0.892 | 8.8 | 68.2 | 28.9 - 91.2 |
| Merchantable stem bark | 9.137 | 0.488 | -7.394 | 0.738 | 15.8 | 6.4 | 2.7 - 9.8 |
| Top wood plus bark | 6.051 | -49.685 | 120.475 | 0.894 | 25.7 | 25.4 | 0.7 - 66.8 |
| | | | White ash (n = 64) | | | | |
| Merchantable stem wood | 93.678 | -6.814 | -56.461 | 0.769 | 13.8 | 64.6 | 27.0 - 90.7 |
| Merchantable stem bark | 11.275 | 2.051 | -8.816 | 0.327 | 30.9 | 86.9 | 2.7 - 19.0 |
| Top wood plus bark | -4.953 | 4.763 | 65.277 | 0.753 | 39.0 | 26.7 | 0.6 - 68.5 |
| | | | Black ash (n = 17) | | | | |
| Merchantable stem wood | 75.421 | 62.257 | -109.466 | 0.741 | 17.5 | 58.2 | 21.9 - 90.2 |
| Merchantable stem bark | 2.200 | 34.020 | -31.739 | 0.295 | 34.3 | 88.2 | 2.5 - 17.3 |
| Top wood plus bark | 22.379 | -96.277 | 141.205 | 0.713 | 36.4 | 33.0 | 1.4 - 74.8 |
| | | | Red ash (n = 24) | | | | |
| Merchantable stem wood | 83.398 | 35.204 | -95.131 | 0.809 | 13.3 | 60.8 | 27.4 - 87.0 |
| Merchantable stem bark | 14.609 | 3.859 | -14.861 | 0.581 | 24.9 | 99.8 | 3.5 - 17.1 |
| Top wood plus bark | 1.993 | -39.063 | 109.992 | 0.798 | 33.7 | 29.2 | 0.1 - 66.1 |
| | | | Basswood (n = 68) | | | | |
| Merchantable stem wood | 77.644 | 56.210 | -108.042 | 0.875 | 10.4 | 62.6 | 19.2 - 91.2 |
| Merchantable stem bark | 10.807 | 21.497 | -27.680 | 0.440 | 27.7 | 11.6 | 3.6 - 22.8 |
| Top wood plus bark | 11.549 | -77.707 | 135.722 | 0.886 | 27.9 | 25.8 | 0.2 - 75.4 |

Table 7. (cont'd)

| Component | Regression coefficients $b_0$ | $b_1$ | $b_2$ | $R^2$ | SEE% | Mean (%) | Range (%) |
|---|---|---|---|---|---|---|---|
| | | | American beech (n = 70) | | | | |
| Merchantable stem wood | 88.296 | 39.002 | -96.680 | 0.831 | 11.8 | 68.9 | 21.0 - 94.5 |
| Merchantable stem bark | 5.838 | 6.056 | -9.870 | 0.455 | 29.1 | 5.2 | 1.5 - 15.5 |
| Top wood plus bark | 5.866 | -45.058 | 106.550 | 0.836 | 33.2 | 25.9 | 0.5 - 77.6 |
| | | | Black cherry (n = 63) | | | | |
| Merchantable stem wood | 94.449 | -9.283 | -53.612 | 0.724 | 14.7 | 66.1 | 23.4 - 93.2 |
| Merchantable stem bark | 11.359 | 0.527 | -7.697 | 0.237 | 39.9 | 84.4 | 2.7 - 23.9 |
| Top wood plus bark | -5.808 | 8.756 | 61.309 | 0.743 | 40.9 | 25.4 | 0.6 - 73.4 |
| | | | White elm (n = 67) | | | | |
| Merchantable stem wood | 92.939 | 1.385 | -66.592 | 0.813 | 12.1 | 63.7 | 22.5 - 91.7 |
| Merchantable stem bark | 11.291 | 2.651 | -10.233 | 0.292 | 37.2 | 83.7 | 1.3 - 20.1 |
| Top wood plus bark | -4.230 | -4.036 | 76.825 | 0.822 | 30.1 | 27.9 | 0.9 - 72.4 |
| | | | Hickory (n = 67) | | | | |
| Merchantable stem wood | 88.439 | 24.672 | -90.829 | 0.791 | 13.8 | 63.2 | 18.3 - 91.9 |
| Merchantable stem bark | 10.927 | 5.022 | -13.047 | 0.489 | 26.7 | 82.5 | 1.6 - 18.1 |
| Top wood plus bark | 0.634 | -29.694 | 103.876 | 0.791 | 34.5 | 28.6 | 0.9 - 79.1 |
| | | | White oak (n = 45) | | | | |
| Merchantable stem wood | 90.538 | -2.508 | -54.275 | 0.776 | 12.9 | 66.0 | 30.8 - 91.4 |
| Merchantable stem bark | 13.653 | -2.516 | -6.611 | 0.331 | 37.2 | 93.1 | 3.0 - 24.2 |
| Top wood plus bark | -4.191 | 5.024 | 60.886 | 0.799 | 37.0 | 24.7 | 0.4 - 66.2 |
| | | | Red oak (n = 100) | | | | |
| Merchantable stem wood | 92.925 | -22.559 | -37.195 | 0.615 | 17.8 | 62.4 | 21.4 - 90.6 |
| Merchantable stem bark | 15.729 | 1.312 | -12.847 | 0.437 | 30.7 | 10.9 | 2.9 - 26.0 |
| Top wood plus bark | -8.654 | 21.247 | 50.042 | 0.650 | 46.7 | 26.7 | 0.1 - 74.1 |
| | | | All hardwoods (n = 1255) | | | | |
| Merchantable stem wood | 84.472 | 33.065 | -91.403 | 0.811 | 13.3 | 62.5 | 13.7 - 94.9 |
| Merchantable stem bark | 10.087 | 12.415 | -18.786 | 0.320 | 37.2 | 9.3 | 1.3 - 39.8 |
| Top wood plus bark | 5.441 | -45.480 | 110.189 | 0.825 | 31.9 | 28.2 | 0.1 - 83.5 |

**Table 8. Regression coefficients and statistics of Equation 4: $OM\% = b_0 + b_1 \cdot (hm/h) + b_2 \cdot (hm/h)^2$**

| Component* | Regression coefficients $b_0$ | $b_1$ | $b_2$ | $R^2$ | SEE% |
|---|---|---|---|---|---|
| | | Trembling aspen (n = 164) | | | |
| Merchantable stem wood | 0.943 | 182.017 | -101.509 | 0.948 | 7.82 |
| Merchantable stem bark | 0.978 | 27.838 | -10.393 | 0.692 | 24.02 |
| Top wood plus bark | 98.079 | -209.855 | 111.902 | 0.964 | 12.51 |
| | | Largetooth aspen (n = 71) | | | |
| Merchantable stem wood | 2.470 | 170.247 | -91.041 | 0.969 | 5.79 |
| Merchantable stem bark | 0.525 | 34.983 | -15.863 | 0.771 | 19.03 |
| Top wood plus bark | 97.005 | -205.230 | 106.904 | 0.990 | 6.64 |
| | | Balsam poplar (n = 87) | | | |
| Merchantable stem wood | 5.821 | 182.087 | -102.280 | 0.925 | 7.9 |
| Merchantable stem bark | 0.757 | 25.443 | -12.849 | 0.588 | 24.9 |
| Top wood plus bark | 93.422 | -207.530 | 115.129 | 0.945 | 17.9 |
| | | White birch (n = 103) | | | |
| Merchantable stem wood | 4.707 | 178.755 | -98.470 | 0.956 | 6.58 |
| Merchantable stem bark | 0.935 | 26.566 | -11.562 | 0.558 | 29.70 |
| Top wood plus bark | 94.358 | -205.321 | 110.032 | 0.962 | 13.73 |
| | | Yellow birch (n = 89) | | | |
| Merchantable stem wood | 2.749 | 209.118 | -129.872 | 0.924 | 7.9 |
| Merchantable stem bark | 2.022 | 20.851 | -9.560 | 0.357 | 36.0 |
| Top wood plus bark | 95.229 | -229.969 | 139.432 | 0.960 | 17.2 |
| | | Sugar maple (n = 89) | | | |
| Merchantable stem wood | 0.923 | 218.456 | -134.802 | 0.931 | 8.1 |
| Merchantable stem bark | -0.162 | 29.482 | -21.276 | 0.464 | 29.1 |
| Top wood plus bark | 99.239 | -247.938 | 156.078 | 0.963 | 16.3 |

*Mean values and ranges of these components are the same as those provided in Table 7.

Table 8. (cont'd)

| Component* | Regression coefficients $b_0$ | $b_1$ | $b_2$ | $R^2$ | SEE% |
|---|---|---|---|---|---|
| | | Red maple (n = 36) | | | |
| Merchantable stem wood | 5.701 | 205.037 | -126.416 | 0.952 | 5.9 |
| Merchantable stem bark | -0.801 | 32.981 | -24.698 | 0.530 | 24.5 |
| Top wood plus bark | 95.100 | -238.018 | 151.114 | 0.971 | 14.0 |
| | | Silver maple (n = 31) | | | |
| Merchantable stem wood | 7.404 | 205.391 | -127.666 | 0.964 | 5.1 |
| Merchantable stem bark | 0.796 | 18.541 | -11.047 | 0.749 | 15.4 |
| Top wood plus bark | 91.800 | -223.932 | 138.713 | 0.962 | 15.5 |
| | | White ash (n = 64) | | | |
| Merchantable stem wood | 4.518 | 191.649 | -109.340 | 0.921 | 8.1 |
| Merchantable stem bark | -0.105 | 29.444 | -18.622 | 0.550 | 25.3 |
| Top wood plus bark | 95.587 | -221.093 | 127.962 | 0.937 | 19.7 |
| | | Black ash (n = 17) | | | |
| Merchantable stem wood | 5.790 | 173.468 | -87.042 | 0.952 | 7.5 |
| Merchantable stem bark | 0.433 | 31.809 | -22.160 | 0.513 | 28.5 |
| Top wood plus bark | 93.777 | -205.277 | 109.202 | 0.965 | 12.7 |
| | | Red ash (n = 24) | | | |
| Merchantable stem wood | 8.626 | 166.532 | -88.121 | 0.893 | 10.0 |
| Merchantable stem bark | -0.002 | 32.111 | -17.312 | 0.749 | 19.3 |
| Top wood plus bark | 91.376 | -198.643 | 105.433 | 0.902 | 23.5 |
| | | Basswood (n = 68) | | | |
| Merchantable stem wood | 2.628 | 186.156 | -104.876 | 0.934 | 7.5 |
| Merchantable stem bark | -0.419 | 40.536 | -26.925 | 0.520 | 25.7 |
| Top wood plus bark | 97.791 | -226.692 | 131.801 | 0.963 | 16.0 |

*Mean values and ranges of these components are the same as those provided in Table 7.

Table 8. (cont'd)

| Component* | Regression coefficients $b_0$ | $b_1$ | $b_2$ | $R^2$ | SEE% |
|---|---|---|---|---|---|
| | | American beech (n = 70) | | | |
| Merchantable stem wood | 3.519 | 209.415 | -123.829 | 0.952 | 6.2 |
| Merchantable stem bark | 0.142 | 16.151 | -9.579 | 0.528 | 27.0 |
| Top wood plus bark | 96.339 | -225.566 | 133.408 | 0.959 | 16.5 |
| | | Black cherry (n = 63) | | | |
| Merchantable stem wood | 3.914 | 197.353 | -115.569 | 0.938 | 7.0 |
| Merchantable stem bark | 0.285 | 27.027 | -17.249 | 0.326 | 37.5 |
| Top wood plus bark | 95.801 | -224.380 | 132.818 | 0.968 | 14.4 |
| | | White elm (n = 67) | | | |
| Merchantable stem wood | 9.608 | 195.956 | -125.377 | 0.891 | 9.3 |
| Merchantable stem bark | 1.641 | 22.800 | -12.361 | 0.384 | 34.7 |
| Top wood plus bark | 88.751 | -218.756 | 137.738 | 0.919 | 20.3 |
| | | Hickory (n = 67) | | | |
| Merchantable stem wood | 8.519 | 182.844 | -104.420 | 0.927 | 8.1 |
| Merchantable stem bark | 0.449 | 26.946 | -16.531 | 0.665 | 21.6 |
| Top wood plus bark | 91.032 | -209.790 | 120.951 | 0.943 | 18.1 |
| | | White oak (n = 45) | | | |
| Merchantable stem wood | 6.179 | 205.137 | -130.539 | 0.897 | 8.7 |
| Merchantable stem bark | -0.400 | 35.019 | -24.484 | 0.364 | 36.2 |
| Top wood plus bark | 94.221 | -240.156 | 155.023 | 0.917 | 23.7 |
| | | Red oak (n = 100) | | | |
| Merchantable stem wood | 5.216 | 185.909 | -109.777 | 0.900 | 9.1 |
| Merchantable stem bark | 0.276 | 32.881 | -17.274 | 0.572 | 26.8 |
| Top wood plus bark | 94.508 | -218.790 | 127.051 | 0.933 | 20.5 |
| | | All hardwoods (n = 1255) | | | |
| Merchantable stem wood | 4.148 | 191.629 | -111.474 | 0.903 | 9.5 |
| Merchantable stem bark | 0.560 | 28.197 | -15.933 | 0.433 | 34.0 |
| Top wood plus bark | 95.292 | -219.826 | 127.407 | 0.943 | 18.2 |

*Mean values and ranges of these components are the same as those provided in Table 7.

**Table 9. Average stump values at 30 cm stump height, as percent of the total stem mass**

| Species | Number of sample trees n | Mean value | | | Stump wood plus bark | |
|---|---|---|---|---|---|---|
| | | Stump wood | Stump bark | Stump wood plus bark | SD* | SE* |
| Trembling aspen | 164 | 3.53 | 0.80 | 4.33 | 1.078 | 0.049 |
| Largetooth aspen | 71 | 3.34 | 0.88 | 4.22 | 0.862 | 0.059 |
| Balsam poplar | 87 | 4.57 | 0.63 | 5.20 | 1.99 | 0.123 |
| White birch | 103 | 5.16 | 0.88 | 6.04 | 1.573 | 0.089 |
| Yellow birch | 89 | 6.51 | 0.72 | 7.23 | 2.521 | 0.154 |
| Sugar maple | 89 | 4.95 | 0.52 | 5.47 | 1.710 | 0.105 |
| Red maple | 36 | 5.92 | 0.60 | 6.52 | 2.613 | 0.251 |
| Silver maple | 31 | 7.00 | 0.55 | 7.55 | 2.472 | 0.256 |
| White ash | 64 | 5.23 | 0.67 | 5.90 | 2.570 | 0.184 |
| Black ash | 17 | 5.88 | 0.77 | 6.65 | 1.844 | 0.258 |
| Red ash | 24 | 5.08 | 0.66 | 5.74 | 2.029 | 0.239 |
| Basswood | 68 | 4.35 | 0.74 | 5.09 | 0.226 | 0.159 |
| American beech | 70 | 5.56 | 0.37 | 5.93 | 2.938 | 0.203 |
| Black cherry | 63 | 4.71 | 0.65 | 5.36 | 2.172 | 0.158 |
| White elm | 67 | 7.30 | 0.89 | 8.19 | 3.145 | 0.222 |
| Hickory | 67 | 5.25 | 0.64 | 5.89 | 2.768 | 0.195 |
| White oak | 45 | 8.03 | 1.11 | 9.14 | 2.860 | 0.246 |
| Red oak | 100 | 6.47 | 1.02 | 7.49 | 2.849 | 0.165 |
| All hardwoods | 1255 | 5.25 | 0.74 | 5.99 | 2.575 | 0.042 |

*SD = standard deviation; SE = standard error of the mean.

**Table 10. Volume (and mass) percentages at different stump heights in relation to stump volume (and mass) at 30 cm**

| Stump height (cm) | % |
|---|---|
| 5 | 17.95 |
| 10 | 35.28 |
| 15 | 52.07 |
| 20 | 68.36 |
| 25 | 84.45 |
| 30 | 100.00 |

**Table 11. Deduction percentages of stump wood and stump bark mass at different stump heights in relation to total stem mass (wood plus bark): an example using white birch**

| Stump height (cm) | Stump wood (%) | Stump bark (%) | Stump wood plus bark (%) |
|---|---|---|---|
| 5 | 0.93* | 0.15 | 1.08 |
| 10 | 1.82 | 0.31 | 2.13 |
| 15 | 2.69 | 0.46 | 3.15 |
| 20 | 3.53 | 0.60 | 4.13 |
| 25 | 4.36 | 0.74 | 5.10 |
| 30 | 5.16 | 0.88 | 6.04 |

*0.93% = (5.16% from Table 9) x (17.95% from Table 10).

**Table 12. Permissible ratios for Equations 3 and 4**

| Species | Equation 3 Smallest permitted dm/d | Equation 4 Largest permitted hm/h |
|---|---|---|
| Trembling aspen | 0.330* | 0.885 |
| Largetooth aspen | 0.318 | 0.842 |
| Balsam poplar | 0.288 | 0.872 |
| White birch | 0.366 | 0.819 |
| Yellow birch | 0.180 | 0.825 |
| Sugar maple | 0.289 | 0.794 |
| Red maple | 0.248 | 0.788 |
| Silver maple | 0.206 | 0.807 |
| White ash | 0.241 | 0.864 |
| Black ash | 0.341 | 0.783 |
| Red ash | 0.293 | 0.798 |
| Basswood | 0.286 | 0.860 |
| American beech | 0.211 | 0.845 |
| Black cherry | 0.245 | 0.845 |
| White elm | 0.262 | 0.794 |
| Hickory | 0.263 | 0.867 |
| White oak | 0.224 | 0.775 |
| Red oak | 0.255 | 0.861 |
| All hardwoods | 0.206 | 0.863 |

*A dm/d ratio of 0.330 means, for example, 7/21.2, 8/24.2, 9/27.3 and 10/30.3, and a value such as 0.100 is not realistic for the species studied.

**Table 13. Percentage distribution of stump, merchantable part and top of the stem by various stump heights for white birch using Equation 3**

| dm/d | Stump height (cm) | Stump wood | Stump bark | Net merchantable stem wood | Net merchantable stem bark | Top wood plus bark | Total |
|---|---|---|---|---|---|---|---|
| | | % of total stem ovendry mass | | | | | |
| 0.40 | 10 | 1.82 | 0.31 | 80.73 | 14.06 | 3.08 | 100.00 |
| | 20 | 3.53 | 0.60 | 79.02 | 13.77 | 3.08 | 100.00 |
| | 30 | 5.16 | 0.88 | 77.39 | 13.49 | 3.08 | 100.00 |
| 0.65 | 10 | 1.82 | 0.31 | 69.27 | 11.25 | 17.35 | 100.00 |
| | 20 | 3.53 | 0.60 | 67.56 | 10.96 | 17.35 | 100.00 |
| | 30 | 5.16 | 0.88 | 65.93 | 10.68 | 17.35 | 100.00 |
| 0.90 | 10 | 1.82 | 0.31 | 37.91 | 5.96 | 54.00 | 100.00 |
| | 20 | 3.53 | 0.60 | 36.20 | 5.67 | 54.00 | 100.00 |
| | 30 | 5.16 | 0.88 | 34.57 | 5.39 | 54.00 | 100.00 |

# APPENDIX F

## An examination of the bias of the whole tree equation

## AN EXAMINATION OF THE BIAS OF THE WHOLE-TREE EQUATION

Because the biomass equations given in this report are only approximations to the true relationships, there will be situations in which they will give biased estimates. This is true irrespective of the form of equation employed, whether a standard linear formation of $d^2h$, a straight line through the origin as in this report, a polynomial, etc.

There are two principal reasons to expect bias when these equations are used to estimate the biomass of a given stand of trees. Firstly, the equations were developed from one specific data set, and may tend to generally overestimate or underestimate biomass when applied to a stand whose characteristics are different. Secondly, even if there is no general tendency for the equations to underestimate or overestimate, they may overestimate for some tree sizes and underestimate for others. This can produce an overall bias, particularly if the stand consists mainly of small trees or mainly of large ones.

Investigation of the first source of bias was deemed irrelevant because of different inherent degrees of bias in each independent data set owing to differences in compositions of the number of trees and of the tree sizes in each set. But the second source could be studied using the original data set, by examining how well the equations fit for all tree sizes. This was done here only for the whole tree equation for all hardwoods combined, to provide a general idea of the fit of the equations. The trees in the data set were divided into size classes, and the average bias was obtained for each class as a percent of the mean biomass for that class as shown in the following table (Note that minus sign indicates underestimates and plus sign overestimates):

| $d^2h$ class | Number of trees | Average bias as a percent of mean OM |
|---|---|---|
| 1 - 1 000 | 195 | -16 |
| 1 001 - 2 000 | 127 | -3 |
| 2 001 - 3 000 | 117 | -1 |
| 3 001 - 4 000 | 91 | +2 |
| 4 001 - 5 000 | 74 | +2 |
| 5 001 - 6 000 | 73 | +3 |
| 6 001 - 7 000 | 79 | -2 |
| 7 001 - 8 000 | 50 | +3 |
| 8 001 - 9 000 | 55 | +2 |
| 9 001 - 10 000 | 52 | 0 |
| 10 001 - 12 000 | 81 | 0 |
| 12 001 - 14 000 | 60 | 0 |
| 14 001 - 16 000 | 67 | +4 |
| 16 001 - 18 000 | 57 | -5 |
| 18 001 - 20 000 | 53 | 0 |
| 20 001 - 30 000 | 124 | -5 |
| 30 001 - 40 000 | 75 | +2 |
| 40 001 - 50 000 | 47 | +4 |
| 50 001 - 60 000 | 24 | -3 |
| 60 000 + | 42 | 0 |
| All combined | 1 543 | -0.36 |

The fit of this equation appears to be fairly good except for the underestimate of 16 percent for the smallest $d^2h$ class. This probably occurs because, for small trees, the 1.30 m measurement height for the diameter is relatively high on the stem instead of being near the base as it is for the other size classes.

A more detailed analysis could be performed for this equation by dividing the trees into diameter classes and also into height classes for each diameter class. This was not done, as it was considered necessary to keep a reasonably large number of trees in each class.

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